

IC/94/225, AS-ITP-94-24, hep-ph/9408242
NEW REVISION June, 1995

On Hadronic Production of the B_c Meson

Chao-Hsi Chang^{+,*,†}, Yu-Qi Chen^{*,†}

Guo-Ping Han[†] and Hong-Tao Jiang[†]

⁺International Center for Theoretical Physics, P.O.Box 586, I-34100 Trieste, Italy.

^{*}CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China.

[†]Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing 100080, China.¹

Abstract

Two of the approaches to the hadronic productions of the double heavy mesons B_c and B_c^* are investigated. Comparison in various aspects on the results obtained by the approaches is made and shown in figures and a table. Some trial understanding of the approaches themselves and the achieved results is presented. The results may be used as some references for discovering the mesons at Tevatron and LHC.

¹Mailing address.

Recently the interest in the B_c meson, one of the double heavy flavor mesons, is aroused widely due to its properties. Similar to the heavy quarkonia, η_c , J/ψ , and η_b , Υ etc., it is a double heavy quark-antiquark bound state, so the QCD inspired potential model will work well for describing the binding effects of it^[1,2]; but different from them, it carries flavors explicitly, so it may decay by weak interaction only, and as a result, it has a much longer lifetime (a typical weak decay one) and plentiful decay channels which have sizable branching ratios^[3-6]. Especially, some of its decays can be calculated quite reliably and may be measurable in the near future^[3,4]. Thus the meson B_c , in addition to the heavy quarkonia, may be used to test the QCD inspired potential models and the acting weak decay mechanisms for relevant heavy flavors further. Another important reason to make the B_c physics interesting, is the study of the B_c meson being accessible soon experimentally. As pointed out by several independent theoretical estimates^[3-12], the cross sections of its production at certain existent and planned colliders are sizable and some typical signals may project over the background.

Having all the possible productions of the double heavy flavored meson reviewed, the authors of refs.[5,6] have pointed out that the most suitable ones of high energy processes to produce sufficient events of the B_c meson at the existent and planned facilities, are those at a Z^0 boson ‘factory’, such as LEP-I, and of energetic hadronic collisions at Tevatron and LHC etc. In ref.[5], besides a complete calculation on the B_c meson production at the level of the lowest order of perturbative QCD (pQCD), the fragmentation functions for $\bar{b} \rightarrow B_c$ and $\bar{b} \rightarrow B_c^*(S\text{-wave})$ were also worked out correctly, while those for $\bar{b} \rightarrow \chi_{(\bar{b}c)}$ ($P\text{-wave}$) in ref.[7]. In fact, it is the first time to work out the fragmentation functions correctly, because not all the terms, being the lowest order, had been taken into account until the authors did. The fragmentation functions obtained by ref.[5] were confirmed by others soon^[8,11]

i.e. the authors of refs.[8,11] recalculated the same fragmentation functions out in an axial gauge, a different gauge from adopted in ref.[5], and the factorization of the fragmentation functions are manipulated more manifestly. Furthermore the evolution of the fragmentation functions with changing of the fragmentation energy scale was also considered in refs.[8,11] by solving the corresponding Altarelli-Parisi equation^[13].

Of all the proposed theoretical estimates^[5–12], besides those of pure phenomenological ones with Monte Carlo simulation such as done in ref.[10], the adopted approaches for estimating the hadronic productions of the B_c meson may be divided into two categories, although they produce results in consistency in order of magnitude (different in values from each other), and all are based on pQCD. The first category is to consider the production in a fragmentation picture i.e. the B_c meson is produced due to fragmentation of a heavy flavor jet (here \bar{b} jet mainly)^[8]. It is very similar to that of a light meson production from a jet, but the fragmentation energy scale is much higher (above that of nonperturbative QCD) that the fragmentation functions are calculated with pQCD. According to pQCD, the production cross section:

$$d\sigma = \sum_{ijk} \int dx_1 \int dx_2 \int dx_3 F_{H_1}^i(x_1, \mu_F) F_{H_2}^j(x_2, \mu_F) \cdot d\hat{\sigma}_{ij \rightarrow kX}(x_1 x_2 x_3, \mu_F) \cdot D_k^{B_c}(x_3, \mu_F), \quad (1)$$

$F_H^i(x, \mu_F)$ is the distribution function of the parton i in the hadron H , $d\hat{\sigma}_{ij \rightarrow kX}(\cdots)$ is the cross section for the relevant jet inclusive production ($i + j \rightarrow k + X$) and $D_k^{B_c}(x, \mu_F)$ is the fragmentation function of B_c from jet k . The formulation here means that the calculation should be carried out at a typical energy scale μ_F of the process. The specific fragmentation functions $D_k^{B_c}(x, \mu_F)$, ($k = \bar{b}, c$) are calculated in the framework of pQCD. In this approach, it is easy to extend straightforwardly up to the leading logarithm approximation (LLA) accuracy level. The second category

is not, as done in the first category, to factorize the “subprocess” $i + j \rightarrow B_c + X$ into two factors further: the ‘jet production’ $i + j \rightarrow k + X$ and the ‘fragmentation’ of the B_c meson from the jet $k \rightarrow B_c + X$, but the subprocess is treated as a whole, and to compute it directly in the framework of pQCD too i.e. the production cross section:

$$d\sigma = \sum_{ijk} \int dx_1 \int dx_2 F_{H_1}^i(x_1, \mu_F) F_{H_2}^j(x_2, \mu_F) d\hat{\sigma}_{ij \rightarrow B_c X}(x_1 x_2, \mu_F). \quad (2)$$

Although the computations of the second category are available only up to the lowest order of pQCD so far, in principle, they may be extended to higher orders with lengthy and boring calculations. In both of the two approaches (at the lowest order approximation), the wave function at original of the $(c\bar{b})$ bound state system will occur in the fragmentation functions and the amplitude of the subprocess $i + j \rightarrow B_c + X$ respectively, whereas the wave function may be obtained from potential model for the double heavy bound state system precisely^[5,7,8,11]. As $B_c^*(1^3S_1)$ meson has a cross section for hadronic productions bigger than that of B_c , and it will decay to the ground state B_c with a branching ratio almost 100% in a very ‘short’ time (without decay vertex in experimental detector) so it contributes the B_c production substantially, thus in the paper we will discuss B_c^* and B_c together from now on.

In hadronic productions of $B_c(B_c^*)$, the substantial contribution is from the subprocess of gluon-gluon fusion $g + g \rightarrow B_c(B_c^*) + b + \bar{c}$ but not from quark-antiquark annihilation $q + \bar{q} \rightarrow B_c(B_c^*) + b + \bar{c}$ at a relatively high energetic colliders such as Tevatron and LHC etc^[6], thus we will restrict ourselves to discuss the subprocess of gluon-gluon fusion and to find out the differences attributed to the adopted approaches from now on in the paper. For convenience, we will denote the first category as Approach-I, whereas the second one as Approach-II.

From the knowledge of pQCD, Approach-I depends more on the factorization

theorem, whereas Approach-II, being much more complicated than Approach-I, is of a fixed order complete calculation. If Approach-I is extended up to the level of LLA, it may achieve better results for very high energy and high P_T problems. Whereas for some of the other problems with not very high energy and very high P_T , the complete fixed order approach, Approach-II even at the lowest order may achieve better ones. We are interested in examining the two categories of the approaches quantitatively for the $B_c(B_c^*)$ production not only due to the experimental interest, but also due to the theoretical interest, because the quantitative results may offer references for study of $B_c(B_c^*)$ experimentally and understanding the production mechanics theoretically.

The precise differences between the two categories in the production of the double heavy meson $B_c(B_c^*)$ in Z^0 decay, have been given in ref.[5], although there the LLA corrections for Approach-I were not considered and the results for the rate of $B_c(B_c^*)$ production were underestimated owing to a smaller QCD coupling ($\alpha_s(Q^2)$ $Q^2 = m_{Z^0}^2$) being adopted. Since the relevant \bar{b} -jet produced in Z^0 decay is very energetic ($E_b = m_{Z^0}/2$ at C.M.S.) and the process is comparatively simple, thus to find the correspondence of the approaches is simple, the difference in values between the approaches in partial width is less than 20%. However, in the hadronic productions the colliding energy of the subprocess varies in a wide region and may be quite ‘low’ at Tevatron, even at LHC, and the subprocess itself is much more complicated. The energies of the subprocess in the hadronic collision at Tevatron, even at LHC, in most cases are much smaller than that in Z^0 decay, and we will return this point more precisely later on. Furthermore the subprocess is much more complicated than that of Z^0 decay: there are 3 diagrams instead of one of Z^0 decay for Approach-I, and there are 36 diagrams instead of 2 for Approach-II. Concerning the hadronic $B_c(B_c^*)$ production, the masses of the heavy quarks inside the meson

$B_c(B_c^*)$ play the role as the proper energy scale in the production, and it is much greater than Λ_{QCD} so pQCD calculations are always applicable, no matter how big P_T the heavy flavor (here the $B_c(B_c^*)$ meson) carries, that is very different from light flavor productions. Generally to know the accurate contributions from the low P_T components quantitatively, which is expected to have substantial different results for the approaches, is interesting, because it is necessary for writing an event generator which may produce reliable low P_T events. To see how low P_T events could be well detected by the concrete detector is interesting for the experiments on the concerned subject(s) i.e. $B_c(B_c^*)$ mesons for present problem, whereas without the reliable event generator, the aim could not be reached at.

The differences between the two categories of the approaches, in fact, may be attributed how to deal with the subprocess of Approach-II. In Approach-II we deal with it as a whole i.e. a complete pQCD calculation on the process, though only the lowest order one is available so far; whereas in Approach-I it is treated to produce heavy quark jets first and then to fragment a meson $B_c(B_c^*)$ from one of the produced heavy jets, thus as known from the proof of the pQCD factorization theorem, Approach-I is not as good as Approach-II even doubtful, if the jet responsible for the fragmentation of a $B_c(B_c^*)$ meson, is not very energetic.

To understand the differences of the approaches, let us analyze the subprocess, $g + g \rightarrow B_c(B_c^*) + b + \bar{c}$, carefully. According to Approach-II, there are 36 Feynman diagrams responsible for it. Some of the typical ones are collected in Fig.1(a,b). It is easy to realize that the diagrams (the amplitude) may be divided (splitted) into 5 independent subgroups (terms) according to their color structure, and each of them alone is gauge invariant^[6]. It is too long to write down here the total amplitude of the subprocess explicitly, however, we may write its color structure out explicitly in a short formulation, and with it we will be able to find out some correspondence

and difference of the two approaches. In general, the formulation for the amplitude:

$$A(a, b, i, j) = \sum_{\alpha=1}^6 C_{\alpha ij}^{ab} M_{\alpha}(\epsilon_1, \epsilon_2, s_1, s_2). \quad (3)$$

Here each of the color factors $C_{\alpha ij}^{ab} (\alpha = 1, 2, \dots, 6)$ is a product of the Gell-Mann matrices:

$$\begin{aligned} C_{1 ij}^{ab} &= (\lambda^c \cdot \lambda^c \cdot \lambda^a \cdot \lambda^b)_{ij} = \frac{N^2 - 1}{N} (\lambda^a \cdot \lambda^b)_{ij} ; \\ C_{2 ij}^{ab} &= (\lambda^c \cdot \lambda^c \cdot \lambda^b \cdot \lambda^a)_{ij} = \frac{N^2 - 1}{N} (\lambda^b \cdot \lambda^a)_{ij} ; \\ C_{3 ij}^{ab} &= (\lambda^c \cdot \lambda^a \cdot \lambda^c \cdot \lambda^b)_{ij} = \frac{-1}{N} (\lambda^a \cdot \lambda^b)_{ij} ; \\ C_{4 ij}^{ab} &= (\lambda^c \cdot \lambda^b \cdot \lambda^c \cdot \lambda^a)_{ij} = \frac{-1}{N} (\lambda^b \cdot \lambda^a)_{ij} ; \\ C_{5 ij}^{ab} &= (\lambda^c \cdot \lambda^a \cdot \lambda^b \cdot \lambda^c)_{ij} = \delta_{ij} \text{tr}(\lambda^a \cdot \lambda^b) - \frac{1}{N} (\lambda^a \lambda^b)_{ij} ; \\ C_{6 ij}^{ab} &= (\lambda^c \cdot \lambda^b \cdot \lambda^a \cdot \lambda^c)_{ij} = \delta_{ij} \text{tr}(\lambda^a \cdot \lambda^b) - \frac{1}{N} (\lambda^b \cdot \lambda^a)_{ij}. \end{aligned} \quad (4)$$

However, we should note here that not all these color factors are independent, because there exists a relation among them, that is

$$C_{3 ij}^{ab} - C_{5 ij}^{ab} = C_{4 ij}^{ab} - C_{6 ij}^{ab}. \quad (5)$$

Therefore only 5 color factors are independent, and we may choose them as:

$$\begin{aligned} C_{m ij}'^{ab} &= C_{m ij}^{ab} \quad (\text{when } m = 1, \dots, 4); \\ C_{5 ij}'^{ab} &= C_{3 ij}^{ab} - C_{5 ij}^{ab}. \end{aligned} \quad (6)$$

Thus the amplitude may be rewritten as:

$$A(a, b, i, j) = \sum_{k=1}^5 C_{k ij}'^{ab} M_k'(\epsilon_1, \epsilon_2, s_1, s_2). \quad (7)$$

Being independent, the coefficients of the color factor $C_{k ij}'^{ab}$, the sub-amplitudes M_k' ($k = 1, 2, \dots, 5$) are individually gauge invariant, thus each of them may

acquire certain meaning. Owing to the fact that each of the amplitudes M'_k is related to certain Feynman diagrams of the 36 precisely, the explicit formulas of M'_k ($k = 1, 2, \dots, 5$) may be written down directly, based on the rules of the dual amplitude method^[8]. Therefore one may find out the correspondences and difference between Approach-I and Approach-II. Since the Approach-I is of a fragmentation of a \bar{b} -quark jet in the $B_c(B_c^*)$ production (the fragmentation of a c -jet contributes too, but it is much less important than that of a \bar{b} -jet), with the decomposition eqs.(4-7) and according to the color structure to trace back to the diagrams, one may find the correspondence: the sub-amplitudes $C'_{k\ ij}{}^{ab}M'_k$ ($k = 1, 2$) is to correspond to some of Approach-I's amplitudes, whereas those of $C'_{k\ ij}{}^{ab}M'_k$ ($k = 3, 4, 5$) cannot find any correspondence in Approach-I. The fact of the correspondences is easy to be understood by means of the Feynmann diagrams of the two approaches: One may find that the diagrams such as Fig.1(a) which contribute to the sub-amplitudes $C'_{k\ ij}{}^{ab}M'_k$ ($k = 1, 2$) substantially in the sense of the color structure, could be understood as if two jets were produced and the fragmentation of $B_c(B_c^*)$ meson was followed, whereas for the diagrams such as Fig.1(b), which contribute to the sub-amplitudes $C'_{k\ ij}{}^{ab}M'_k$ ($k = 3, 4, 5$) substantially, there is no similar correspondence at all in the above sense to Approach-I. Therefore we expect the results achieved by the two approaches being different, so a thorough investigation of the approaches quantitatively, even though numerically, is interesting. We will devote this paper to the investigation², i.e. to compare the $B_c(B_c^*)$ hadronic productions of the approaches quantitatively in various aspects. We will plot the numerical results of each observable, obtained by the two approaches into one figure together, different figures show different aspects of the approaches, and finally we will try to reach at some

² During the period of revising the paper, several papers^[19,20] come out and certain disagreements on the calculations are presented, thus to clarify the situation is also necessary.

conclusions.

First of all, we calculate the total cross sections of the $B_c(B_c^*)$ productions by the two approaches at various hadronic colliders i.e. for various C.M.S's energies of the colliding hadrons, but only the lowest order for the subprocess $g+g \rightarrow B_c(B_c^*)+b+\bar{c}$, is concerned. The obtained total cross sections are put into Tab.1. We should note here that throughout the paper without special statement, the following manipulations and parameters are taken. When calculating the productions of $p-p$ and $p-\bar{p}$ collisions, only gluon-gluon fusion mechanism is considered due to its domination over the others such as quark-antiquark etc^[6]; the CTEQ3 structure functions with $\Lambda_{\overline{MS}}^{QCD}(n_f=4)=0.239\text{GeV}$ (corresponding $\alpha_s(m_Z^2)=0.112$)^[14] and $\alpha_s(Q^2)$ with an energy scale $Q^2=\bar{s}/4$ (\bar{s} is the c.m. energy squared of the subprocess) are adopted. As for the masses, the values $m_c=1.5\text{GeV}$, $m_b=4.9\text{GeV}$ and $M_{B_c(B_c^*)}=6.4\text{GeV}$ ^[2] are taken. Furthermore in the calculations the wave functions of B_c and B_c^* at origin are obtained from potential model and the difference of wave functions, as the masses, for B_c and B_c^* is ignored here for the 'lowest order calculation'. In order to compare with those adopted in literature easily, what we adopt here is in decay constant formulation: $f_{B_c} \simeq 480\text{MeV}$ (under the convention $f_\pi=132\text{MeV}$).

Note here that in the table when calculating the subprocess $g+g \rightarrow B_c(B_c^*)+b+\bar{c}$ at $\sqrt{\bar{s}}=20, 30, 60\text{GeV}$, a constant of strong coupling $\alpha_s=0.2$ is taken, and when the row is denoted with a '*' ('**'), the results are indicated to have a cut for small $P_T \leq 5\text{GeV}$ ($P_T \leq 10\text{GeV}$). Up to the concerned order of pQCD, the uncertainties here come only from the choices of the values of m_c , m_b , $M_{B_c(B_c^*)}$, α_s and f_{B_c} .

TABLE I. The total cross sections for the productions of the B_c meson and its excited state B_c^* obtained by the two approaches (in unit nb).

Collision	Approach-I		Approach-II	
	$B_c (1^1S_0)$	$B_c^* (1^3S_1)$	$B_c (1^1S_0)$	$B_c^* (1^3S_1)$
$p\bar{p}(\sqrt{S} = 1.8TeV)$	0.747(4)	1.23(1)	0.850(8)	2.07(2)
$p\bar{p}(\sqrt{S} = 1.8TeV, *)$	0.229(2)	0.389(3)	0.259(4)	0.646(6)
$p\bar{p}(\sqrt{S} = 1.8TeV, **)$	0.0331(9)	0.0570(6)	0.0373(1)	0.0894(3)
$pp(\sqrt{S} = 14TeV)$	8.63(5)	14.0(1)	10.6(1)	26.4(3)
$pp(\sqrt{S} = 14TeV, *)$	3.07(3)	5.11(4)	3.71(6)	9.43(9)
$pp(\sqrt{S} = 14TeV, **)$	0.584(7)	0.986(10)	0.698(1)	1.69(4)
$gg(\sqrt{s} = 20GeV)$	$0.704(5) \cdot 10^{-2}$	$0.118(1) \cdot 10^{-1}$	$0.661(7) \cdot 10^{-2}$	$0.160(2) \cdot 10^{-1}$
$gg(\sqrt{s} = 30GeV)$	$0.678(8) \cdot 10^{-2}$	$0.103(1) \cdot 10^{-1}$	$0.949(8) \cdot 10^{-2}$	$0.244(3) \cdot 10^{-1}$
$gg(\sqrt{s} = 60GeV)$	$0.321(7) \cdot 10^{-2}$	$0.456(9) \cdot 10^{-2}$	$0.782(9) \cdot 10^{-2}$	$0.203(3) \cdot 10^{-1}$

The P_T dependence of the productions at various colliders Tevatron and LHC is interesting experimentally, thus we have calculated it and plotted the results in Fig.2. In the calculations, the low P_T component contribution has been taken into account too, though for Approach-I the computation is problematic. It is because the production closing to the threshold (where P_T cannot be big) needs special consideration and corrections in Approach-I, but here we merely make the ‘approximation’: the P_T of $B_c(B_c^*)$ being fixed in the direction of the produced heavy quark jet, in fact, it is not a good approximation when the ‘fragmentation’ is very close to the threshold of the $B_c(B_c^*)$ meson production, thus the low P_T component contribution as shown in Fig.2 is not so well estimated for Approach-I. From the figure one may see that for the B_c production, the difference between the two approaches is not sizable but for the B_c^* production it is quite great (about a factor two even greater) and, general speaking, as P_T is going high the production cross sections predicted by the two approaches are approaching to equal (for B_c^* , up to $P_T = 20GeV$ they are still different).

In order to have an outline about the gluon-gluon subprocess in hadronic colli-

sions, in Fig.3, we present the production cross sections at Tevatron and LHC versus the collision energy \bar{s} of the gluons inside the collision hadrons. As the small P_T component of the productions is not able to measure, we have imposed a cut for those of small $P_T(\leq 5GeV)$ here. From the figure, one may see the cross sections drop in a logarithm scale versus \bar{s} increasing. In fact, if we had not imposed the cut for small P_T , the cross sections would have a “peak” around $20GeV$ (not very far from the threshold of the subprocess $\sqrt{\bar{s}} \sim 12.8GeV$), and then would drop ³. One may see that when \bar{s} reaches at $80GeV$, the cross sections have dropped down at least one more orders of the magnitude already. Thus one may conclude that in the hadronic collisions the dominant contribution to the B_c and B_c^* meson productions is not from very high energetic gluon fusion but from relatively low energy, that is the great difference from that in Z^0 decay as we emphasized earlier in the paper. For B_c^* production, the cross sections obtained by Approach-II are greater than those by Approach-I at various energies with a factor 5 or greater, but for B_c production, the difference caused by the two approaches is within a factor 2, less than that for B_c^* production.

We should note here that besides the cut for small P_T being imposed and the coupling constant α_s being running, the cross sections in Fig.3 are different in meaning from that of the gluon-gluon fusion for precise $\sqrt{\bar{s}}$ in Tab.1, as the later is merely of gluon-gluon fusion but the former has the structure functions of the collision hadrons convoluted into.

All the resultant cross sections of the hadronic productions are achieved always by a convolution of the cross section of the relevant subprocess and a common factor, the structure functions of the incoming hadrons of the collisions. In order to highlight

³To shorten the paper and to present the more useful results, we would not present the curves without P_T cut here, although we have them.

the differences of the two approaches, we have also calculated the subprocess cross sections as if the subprocess is an independent one, i.e. the cross sections of the gluon-gluon fusion at various precise energies. The total cross sections have been put in Tab.1 already, but the transverse momentum P_T and rapidity Y distributions at various C.M.S's energies are presented in Figs.4: in Fig.4(a,b) for $\sqrt{s} = 30 \text{ GeV}$, Fig.4(c,d) for $\sqrt{s} = 60 \text{ GeV}$ respectively. In these calculations, we have taken $\alpha_s = 0.2$, a constant, as emphasized above. One may see the fact very clearly that, as expected, the values obtained by Approach-II at small P_T and small Y are always greater than those obtained by Approach-I, whereas the values are approaching close when P_T or Y increases.

In summary, the two approaches cause some substantial differences in total cross sections and the P_T distribution etc. indeed, especially at low P_T . Approach-II should be suitable at low P_T , even at low P_T and low Y both. The heavy masses of the quarks inside the meson play the role to offer the least and proper energy scale in the concerned productions and to guarantee pQCD being applicable always. Namely it is the heavy quark masses being the least proper energy scale, instead of Λ_{QCD} , that appear in the formulas (appear in α_s , the coupling constant for the lowest order calculations and in the logarithmically large terms if higher order calculations are carried out). Furthermore, the most important productions of the $B_c(B_c^*)$ meson are shown in Fig.3 not due to very energetic parton collisions, and from Fig.2 one may also see that the P_T cannot be great in the interesting processes concerned in the paper, thus in order to collect as many as possible events of the mesons B_c and B_c^* so as to discover them and to study their properties, one could not estimate the low P_T components of the productions too roughly from very beginning and should try to have a good one as one can. For this purpose, it is sure that Approach-II is good, and the logarithmical terms to the heavy quark masses need not to worry about

too much, as the terms appear from high order calculations and become important at very large P_T only^[16]. Approach-I is better than Approach-II for estimating the productions at very large P_T if the former taking into account the large logarithmical terms to the heavy quark masses by LLA but the later not (as the present case). Whereas for the concerning hadronic productions at Tevatron, even at LHC, the former's advantages have not "matured" yet because of the same reason as pointed out above: P_T cannot be very great at the concerned processes.

We think that the differences of the obtained results by the two approaches may be understood by the fact that Approach-II involves more mechanisms than those Approach-I does, as argued in terms of eqs. (3-7) early. Recently the authors of ref.[17] also recognized that certain higher order gluon fragmentation besides the fragmentation of a heavy quark may contribute to B_c production substantially.

According to the experiences of heavy quark productions in hadronic collisions and the theoretical loop calculations, we know that a full perturbative QCD calculation up to one loop level may achieve quite high accuracy^[18], thus a higher order full perturbative QCD calculation on the hadronic production of the double heavy flavor meson $B_c(B_c^*)$ under Approach-II will be very interesting surely^[16].

In the procedure of revising the paper, several calculations^[19-21] on the same problems appear. The authors of ref.[20] have found that in their earlier version they had omitted a color factor $1/\sqrt{3}$ in amplitude, and when having the mistake corrected they have found a nice agreement between theirs and those of ref.[6]. The authors of ref.[19] have investigated various calculations quite systematically, therefore we have checked the numerical results for the subprocess $g+g \rightarrow B_c(B_c^*) + b + \bar{c}$ at $\sqrt{s} = 20, 40 GeV$, by means of our program but having their parameters. As a result, we have found that our results and theirs are consistent with each

other exactly within the Monte Calor errors⁴, that in fact is a confirmation for our programs and those of ref.[19]. Only for updating, the new version of the structure functions CTEQ3^[14] and the parameters appearing in the calculations as quoted above (e.g. the energy of HLC $\sqrt{S} = 14GeV$ etc), which are slight different from those of ref.[19] even ref.[6], have been adopted here so that the numerical results for the total cross sections and the other obseables involving two structure functions of the two colliding hadrons are reasonably different from those of refs.[6,19,20] a little. However there are some of disagreements between our results and those of ref.[21].

Acknowledgements

The work was supported in part by the National Natural Science Foundation of China and the Grant LWTZ-1298 of Chinese Academy of Science. The authors would like to thank Prof. Yuan-Ben Dai for useful discussions. One of the author (Chao-Hsi Chang) would like to thank Profs. A. Salam and S. Randjbar-Daemi, the International Atomic Energy Agency and UNESCO for hospitality during his stay at ICTP. Finally the authors would like to thank Dr. H.L. Lai for offering the numerical program of the structure functions and the referees for useful suggestions.

⁴In the early version of this paper, there was a mistake in the programs for the numerical calculations, thus there were some disagreements between our earlier results and those of ref.[19].

References

- [1] J. Richardson, Phys. Lett. **B82** (1979) 272;
W. Buchmüller and S.-H.H. Tye, Phys. Rev. **D24** (1981) 132.
- [2] S. Godfrey and N. Isgur, Phys. Rev. **D32** (1985) 189.
W. Kwong and J. L. Rosner, Phys. Rev. **D44** (1991) 212;
Yu-Qi Chen and Yu-Ping Kuang, Phys. Rev. **D46** (1992) 1165;
E. Eichten and C. Quigg, Phys. Rev. **D49** (1994) 5845;
V.V. Kiselev, A.K. Likhoded and A.V. Tkabladze, Preprint IHEP 94-51, hep-ph/9406339.
- [3] Chao-Hsi Chang and Yu-Qi Chen, Phys. Rev. **D49** (1994) 3399; Preprint AS-ITP-93-80.
- [4] P. Colangelo, G. Nardulli and N. Paver, Z. Phys. **57** (1993) 43;
S.S. Gershtein, A.K. Likhoded and S.R. Slabospitsky, Intern'l. J. Mod. Phys. **6** (1991) 2309.
- [5] Chao-Hsi Chang and Yu-Qi Chen, Phys. Rev. **D46** (1992) 3845; Erratum, Phys. Rev. **D50** (1994) 6013; Phys. Lett. **B284** (1992) 127.
- [6] Chao-Hsi Chang and Yu-Qi Chen, Phys. Rev. **D48** (1993) 4086.
- [7] Yu-Qi Chen, Phys. Rev. **D48** (1993) 5181.
- [8] E. Braaten, K. Cheung and T. C. Yuan, Phys. Rev. **D48** (1993) 4230; R5049.
- [9] K. Cheung, Phys. Rev. Lett. **71** (1993) 3413.
- [10] M. Lusignoli, M. Masetti and S. Petrarca, Phys. Lett. **B266** (1991) 142.
- [11] S.R. Slabospitsky, IHEP 94-53, hep-ph/9404346.

- [12] V.V. Kiselev et al., Sov. J. Nucl. Phys. **49** (1989) 1100;
V.V. Kiselev, A.K. Likhoded and M.V. Shevlyagin, Preprint IHEP95-58, hep-ph/9401348.
- [13] G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298.
- [14] H.L. Lai, et al. Preprint MSU-HEP-41024, CTEQ-404, hep-ph/9410404.
- [15] V. Barger, A.L. Stange and R.J.N. Phillips, Phys. Rev. **D44** (1991) 1987.
- [16] Higher order calculations are undertaken, and preprint in preparation.
- [17] K. Cheung and T. C. Yuan, Preprint NUHEP-TH-93-30, UCD-93-35.
- [18] P. Nason, S. Dawson and R.K. Ellis, Nucl. Phys. **B303** 607 (1988); **327** 49 (1989).
- [19] K. Kolodziej, A. Leike and R. Rückl, Preprint MPI-PhT/95-36, LMU-08/95, hep-ph/9505298.
- [20] A.V. Berezhnoy, A.K. Likhoded and O.P. Yushchenko, IHEP-95-59, hep-ph/9504302.
- [21] M. Masetti and F. Sartogo, hep-ph/9503491.

Figure Captions

1. Some Typical Feymann Diagrams for the Subprocess $g + g \rightarrow B_c(B_c^*) + b + \bar{c}$.
(Here the spots in the figure denote the bound state $B_c(B_c^*)$. In fact, there are 36 diagrams in total, but here we plot some of them only for illustrating the existence of various mechanisms involved in the process.)
 - (a) The Feymann Diagrams Correspond to that for Fragmentation mechanism mainly.
 - (b) The Feymann Diagrams Correspond to those for Some else besides the Fragmentation Mechanism mainly.
2. The the Transverse Momentum P_T Distribution for $B_c(B_c^*)$ Productions at Tevatron and LHC for Approach-I and Approach-II.
 - (a) The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the the Transverse Momentum P_T at Tevatron. The solid line ($A V$) and the dashed line ($C V$) are those of the B_c^* productions obtained by Approach-I and Approach-II respectively; The dashed-dotted line ($A P$) and the dotted line ($C P$) are those of the B_c productions obtained by Approach-I and Approach-II respectively.
 - (b) The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the the Transverse Momentum P_T at LHC. The notation of the lines is the same as Fig.2a.
3. The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the C.M. Energies of the Colliding Gluon-Gluon at the Colliders for Approach-I and Approach-II.

- (a) The Differential Cross Sections of the $B_c(B_c^*)$ Production versus the C.M. Energies of the Colliding Gluon-Gluon at Tevatron. The notation of the lines is the same in Fig.2a.
 - (b) The Differential Cross Sections of the $B_c(B_c^*)$ Production versus the C.M. Energies of the Colliding Gluon-Gluon at LHC. The notation of the lines is the same in Fig.2a.
4. The Distributions of the Transverse Momentum P_T and the Rapidity Y for $B_c(B_c^*)$ Productions at Precise Energies (\sqrt{s}) of the Gluon-Gluon Collisions Respectively.
- (a) The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the Transverse Momentum P_T at $\sqrt{s} = 30\text{GeV}$. The notation of the lines is the same in Fig.2a.
 - (b) The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the Rapidity Y at $\sqrt{s} = 30\text{GeV}$. The notation of the lines is the same in Fig.2a.
 - (c) The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the Transverse Momentum P_T at $\sqrt{s} = 60\text{GeV}$. The notation of the lines is the same in Fig.2a.
 - (d) The Differential Cross Sections of the $B_c(B_c^*)$ Productions versus the Rapidity Y at $\sqrt{s} = 60\text{GeV}$. The notation of the lines is the same in Fig.2a.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9408242v3>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9408242v3>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9408242v3>

This figure "fig1-4.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9408242v3>